2/EH-29 (ii) (Syllabus–2015)

2016

(April)

MATHEMATICS

(Elective/Honours)

SECOND PAPER

(Geometry and Vector Calculus)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each unit

UNIT—I

1. *(a)* Show that the equation

$$4x^2+12xy+9y^2+8x+12y=0$$

represents a pair of parallel lines and find the distance between them. 5

(b) If the straight lines represented by the equation

$$x^2(\tan^2\phi+\cos^2\phi)-2xy\tan\phi+y^2\sin^2\phi=0$$

make angles α and β with the axis of x, then show that $\tan\alpha-\tan\beta=2$. 5

(c) (i) Find the equation of the diameter of the conic $4x^2+6xy-5y^2=1$ conjugate to the diameter $y=2x$.

(ii) Find the asymptotes of the hyperbola $xy+4x+3y+5=0$. 3+2=5

2. (a) Find the equation of the tangent to the conic $4x^2+3xy+2y^2-3x+5y+7=0$ at the point (1, − 2). 5

(b) If by transformation from one set of rectangular axes to another with the same origin the expression $ax+by$ changes to $a'x'+b'y'$, then prove that

$$a^2+b^2=a'^2+b'^2$$ 5

(c) Find the vertex and the length of the latus rectum of the parabola

$$(3x+4y-17)^2=35(4x-3y-6)$$ 5

UNIT—II

3. (a) Show that the tangent to an ellipse at either extremity of a diameter is parallel to the system of chords bisected by the diameter. 5

(b) Prove that if the straight line $\lambda x+\mu y+\nu=0$ touches the parabola $y^2-4px+4pq=0$, then

$$\lambda^2 q+\lambda\nu-p\mu^2=0$$ 5

(c) Find the locus of the point whose distance from the origin is equal to its perpendicular distance from the plane $2x+3y-6z=9$. 5

4. (a) Find the equation of the plane passing through the middle point of the join of the points (2, − 3, 1) and (4, 5, − 3), and is perpendicular to the line joining the points. 5

(b) If p and p' are the lengths of the two segments of any focal chord of the parabola $y^2=4x$, then show that $p+p'=pp'$. 5

(c) Show that the normal to the rectangular hyperbola $xy=c^2$ at the point t meets the curve again at the point t' such that $t^3t'=-1$. 5

UNIT—III

5. (a) Prove that the lines

$$\frac{x-2}{4}=\frac{y+1}{3}=\frac{z-3}{5}$$

and

$$x+2y+3z-9=0=2x-y+2z-11$$

are coplanar. 5

(b) Find the equation of the cylinder whose generators are parallel to the line

$$\frac{x}{2}=\frac{y}{5}=\frac{z}{3}$$

and which passes through the origin $z=0,\ 3x^2+4y^2=12$. 5

(c) (i) Find the centre and radius of the sphere given by

$$x^2+y^2+z^2+3x-4y+5z+5=0$$

(ii) Find the equation of the tangent plane to the sphere

$$x^2+y^2+z^2=14$$

at the point (1, − 2, 3). 2+3=5

6. (a) Find the SD between the y-axis and the line

$$\frac{x-1}{5}=\frac{y-7}{-4}=\frac{z+3}{12}$$

5

(b) Obtain the equation of the sphere having circle

$$x^2+y^2+z^2+10y-4z-8=0,$$

$$x+y+z=3$$

as the great circle. 5

(c) Find the equation of the cone whose vertex is the point (1, 1, 0) and whose guiding curve is $y=0,\ x^2+z^2=4$. 5

UNIT—IV

7. (a) Interpret $\vec{a}\times\vec{b}$ geometrically, where $\vec{a}$ and $\vec{b}$ are non-zero vectors. 5

(b) The position vectors of three points A, B and C are $A(1, 1, -2)$, $B(0, -2, 2)$ and $C(1, 4, -6)$. Find the length of the perpendicular from B to AC. 5

(c) If $\vec{r}=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$ and $\hat{s}=\sin t\hat{i}-\cos t\hat{j}$, then find $\frac{d}{dt}(\vec{r}\cdot\vec{s})$. 5

8. (a) Show that

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a}\,\vec{b}\,\vec{d}]\vec{c}-[\vec{a}\,\vec{b}\,\vec{c}]\vec{d}$$

4

(b) Show that the points $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+4\hat{j}+4\hat{k}$ and $4(\hat{i}+\hat{j}+\hat{k})$ are coplanar. 5

(c) Show that a necessary and sufficient condition that a proper vector $\vec{u}$ always remains parallel to a fixed line is that

$$\vec{u}\times\frac{d\vec{u}}{dt}=\vec{0}$$ 6

UNIT—V

9. *(a)* If $r=|\vec{r}|$, then show that

$$\vec{\nabla}f(r)=\frac{f'(r)}{r}\vec{r}$$ 5

(b) Find the velocity and acceleration of a particle which moves along the curve $x=2\sin 3t$, $y=2\cos 3t$ and $z=8t$ at any time t. 5

(c) Find the directional derivative of $f=xy+yz+zx$ in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$ at the point (1, 2, 0). 5

10. *(a)* Show that $\vec{\nabla}\times\vec{\rho}=\vec{0}$, where $\vec{\rho}=m\frac{\vec{r}}{r^3}$, m is constant. 5

(b) Show that $\nabla^2\left(\frac{1}{r}\right)=0$, where $r=|\vec{r}|$ and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. 5

(c) Find the equations of tangent plane to the surface $x^2+y^2-z=0$ at the point $(2, -1, 5)$. 5

★ ★ ★

2/H–76 (v) (a) (Syllabus–2015)

2 0 1 7

(April)

COMMERCE

(Honours)

(Fundamental Mathematics)

(BC–202)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer *any* **five** questions

1. (a) Express $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix. 5

(b) Compute the inverse of

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ 5

(c) Solve the following equations by Cramer's rule : 5

$$4x - y - z = 1$$
$$x + y - 2z = 1$$
$$3x + 2y + z = 8$$

2. *(a)* Find the domain and the range of the following functions : 3+3

(i) $\frac{1}{\sqrt{x-3}}$

(ii) $\frac{x}{|x|}$

(b) A vegetable seller charges ₹ 25 per papaya for purchase of 6 or less papayas and charges ₹ 20 per papaya if the purchase is more than 6 or less than 11 papayas. If the purchase is more than 11 or equal to 11, he charges ₹ 10 per papaya. Find the cost function $C(x)$ where $C(x)$ represents cost of buying x papayas. 4

(c) If $f(x) = ax + b$ such that $f(-1) = 5$ and $f(1) = 6$, find $f(x)$. Also find the value of $f(10)$. 3+2

3. *(a)* Evaluate the following limits : 3+4

(i) $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$

(ii) $\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1-3x}}{x}$

(b) A function $f(x)$ is defined by

$$f(x) = \begin{cases} -x, & \text{when } x \le 0 \\ x, & \text{when } 0 < x < 1 \\ 2-x, & \text{when } x \ge 1 \end{cases}$$

Show that $f(x)$ is continuous at $x = 0$ and $x = 1$. 5

(c) Draw the graph of the following function : 3

$$f(x) = \begin{cases} -3x, & \text{when } x < 0 \\ x, & \text{when } 0 \le x \le 2 \\ 2, & \text{when } 2 < x \le 4 \end{cases}$$

4. *(a)* Find the first-order derivative of the following functions (any *two*) : 4×2=8

(i) $f(x) = \frac{e^x \log_e x}{x^2}$

(ii) $f(x) = x^x$

(iii) $f(x) = \sqrt{2 + \sqrt{2 + x}}$

(b) Find the elasticities of demand and supply at equilibrium price for the demand function $p = 16 - x^2$ and supply function $p = 2x^2 + 4$, where p is price per unit output and x is the output. 7

5. *(a)* A firm has total revenue function $R(x) = 100x - x^2$ and total cost function $C(x) = x^3 - \frac{57}{2}x^2$, where x is the level of output. Determine the maximum profit. 5

(b) A central agency needs to determine the rent to charge for each of the 200 apartments in order to attain maximum income. Experience shows that if the rent is set at ₹ 150 a week, all units are occupied but for each ₹ 5 per week increase in rent, 5 units become vacant. What rent should be fixed to maximize revenue? Determine also the maximum revenue. 6

(c) The side of an equilateral triangle is 5 cm and is increasing at the rate of $\sqrt{3}$ cm/sec. How fast is—

(i) the area increasing;

(ii) the perimeter increasing?

(Given $A = \frac{\sqrt{3}}{4} a^2$, where a = side and A = area) 4

6. (a) A company expects cash inflows from its investment proposal, it has undertaken in time period zero, ₹ 2,00,000 in the first year and ₹ 1,50,000 in the second year and expects ₹ 1,00,000 for the next eight years. What would be the present value of cash inflows, assuming a 20% rate of interest? 7

(b) If your expected rate of return on investment is 12% per annum and you find a 10% debenture in the market at ₹ 850, would you buy the debenture? Given that the maturity period of the debenture left is 4 years and the maturity value is ₹ 1,000. 8

7. (a) A bond is available for ₹ 1,500 it offers, including one immediate payment and 10 annual payments of ₹ 200. Find the rate of return on the bond. 4

(b) The annual increase of population of a village is 3% approximately. The population of the city was 30 lakhs at the end of 2015. What will be the population at the end of 2030? 4

(c) A machine costs ₹ 5,00,000 with a working life of 5 years and a scrap value of ₹ 1,00,000 at the end. Calculate the yearly depreciation as per written down value method and straight-line method if the rate of depreciation is 10%. 7

★★★

2017

(April)

MATHEMATICS

(Elective / Honours)

(Geometry and Vector Calculus)

(GHS–21)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. *(a)* If the two pairs of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, prove that $pq + 1 = 0$. 5

(b) Find the angle through which a set of rectangular axes must be turned without the change of origin so that the expression $7x^2 + 4xy + 3y^2$ will be transformed into the form $a'x^2 + b'y^2$. 5

(c) Find the diameter of the conic $15x^2 - 20xy + 16y^2 = 1$ conjugate to the diameter $y + 2x = 0$. 5

2. (a) Find the lengths of the semiaxes of the conic $ax^2 + 2hxy + ay^2 = d$. 6

(b) Find the centre of the conic given by the equation

$$3x^2 - 8xy + 7y^2 - 4x + 2y - 7 = 0$$ 5

(c) Find the equation of the polar of the point (2, 3) with respect to the conic

$$x^2 + 3xy + 4y^2 - 5x + 3 = 0$$ 4

UNIT—II

3. (a) Prove that the locus of the point of intersection of the normals to the parabola $y^2 = 4ax$ at the extremities of a focal chord is the parabola $y^2 = a(x - 3a)$. 5

(b) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant. 5

(c) Prove that the locus of the middle point of the portion of a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

intercepted between the axes is given by

$$\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$$ 5

4. (a) If e_1 and e_2 be the eccentricities of a hyperbola and its conjugate, show that

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$ 5

(b) Prove that two tangents can be drawn from a given point of an ellipse. 5

(c) If the tangent $y = mx + \sqrt{a^2m^2 - b^2}$ touches the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point $(a\sec\theta, \ b\tan\theta)$, prove that

$$\sin\theta = \frac{b}{am}$$ 5

UNIT—III

5. (a) Find the equation of the plane passing through the point (1, −2, 1) and the line of intersection of the planes

$2x - y + 3z - 2 = 0$ and $x + 2y - 4z + 3 = 0$ 4

(b) Prove that the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and}$$

$$4x - 3y + 1 = 0 = 5x - 3z + 2$$

are coplanar. 5

(c) Prove that the shortest distance between the lines

$$\frac{x-5}{3} = \frac{y-7}{-16} = \frac{z-3}{7} \text{ and}$$

$$\frac{x-9}{3} = \frac{y-13}{8} = \frac{z-15}{-5}$$

is 14. 6

6. (a) Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 = 21$ at the point (1, −2, 4) and passes through the point (3, 4, 0). 5

(b) Find the equation of the cone whose vertex is (2, 2, 2) and the base is $z = 0$, $x^2 + y^2 = 36$. 5

(c) Find the equation of a right circular cylinder whose axis is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$$

and its radius is 5. 5

UNIT—IV

7. (a) If $\hat{a}, \hat{b}, \hat{c}$ be three unit vectors such that

$$\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$$

find the angles which $\hat{a}$ makes with $\hat{b}$ and $\hat{c}$, given that $\hat{b}$ and $\hat{c}$ being non-parallel. 5

(b) Prove that

$$(\vec{b} \times \vec{c}) \cdot (\vec{a} \times \vec{d}) + (\vec{c} \times \vec{a}) \cdot (\vec{b} \times \vec{d}) + (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 0$$ 5

(c) Prove the Lagrange's identity 5

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix}$$

8. (a) Show that a necessary and sufficient condition for $\vec{u}(t)$ to be constant is

$$\frac{d\vec{u}}{dt} = 0$$ 3

(b) If $\hat{r}$ is a unit vector, show that

$$\left| \hat{r} \times \frac{d\hat{r}}{dt} \right| = \left| \frac{d\hat{r}}{dt} \right|$$ 3

(c) If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, show that

(i) $\hat{r} \times \frac{d\vec{r}}{dt} = \omega\vec{a} \times \vec{b}$

(ii) $\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$

where $\vec{a}$ and $\vec{b}$ are constant vectors. 2+3

(d) A particle moves along the curve $x = 4\cos t$, $y = 4\sin t$, $z = 6t$. Find the velocity and acceleration at time $t = 0$ and $t = \frac{\pi}{2}$. 4

UNIT—V

9. (a) Find the directional derivative of the function

$$f(x) = x^2 - y^2 + 2z^2$$

at the point $P(1, 2, 3)$ in the direction of the line PQ, where Q has coordinates $(5, 0, 4)$. 5

(b) Show that $\text{grad} f(r) \times \vec{r} = 0$, where

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$ 5

(c) Determine the constants a, b, c so that the vector

$$\vec{f} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

is irrotational. 5

10. (a) Find a unit normal to the surface $\phi = 2x^2y + 3yz - 4$ at the point $(1, -1, -2)$. 5

(b) If $r = |\vec{r}|$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $\nabla \log |\vec{r}| = \frac{1}{r^2}\vec{r}$. 5

(c) Find the divergence and curl of the vector

$$\vec{f} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$$ 5

★★★

2 0 1 8

(April)

MATHEMATICS

(Elective/Honours)

(Geometry and Vector Calculus)

(GHS–21)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. *(a)* If by rotation of the rectangular axes the equation $17x^2 + 18xy - 7y^2 = 1$ reduces to the form $ax^2 + by^2 = 1$, find the angle through which the axes are rotated. Also find the values of a and b. 4

(b) Prove that the equation

$$2x^2 + xy - 6y^2 - 6x + 23y - 20 = 0$$

represents a pair of straight lines. Find the coordinates of their point of intersection. 5

(c) Reduce the equation

$$17x^2 + 12xy + 8y^2 - 46x - 28y + 17 = 0$$

to the standard form. 6

2. (a) Prove that the equation of the tangent to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy = 0$$

at the origin is $gx + fy = 0$. 6

(b) Find the diameter of the conic

$$15x^2 - 20xy + 16y^2 = 1$$

conjugate to the diameter $y + 2x = 0$. 5

(c) Prove that the two hyperbolas

$$4x^2 + 3xy + 5x + 21 = 0$$

and $\quad x^2 - 4xy - 3x + 19 = 0$

have a common asymptote. Also find the other asymptotes. 4

UNIT—II

3. (a) Show that the locus of the point of intersection of any two perpendicular tangents to the parabola $y^2 = 4ax$ is the directrix. 5

(b) Find the asymptotes of the hyperbola

$$xy + ax + by = 0$$ 5

(c) A tangent to the parabola $y^2 = 8x$ makes an angle of 45° with the straight line $y = 3x + 5$. Find the equation of the tangent and its point of contact. 5

4. (a) Show that the normal to the rectangular hyperbola $xy = c^2$ at the point t meets the curve again at the point t' such that $t^3 t' = -1$. 5

(b) Prove that the straight line $\frac{ax}{3} + \frac{by}{4} = c$ will be a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

if $5c = a^2 e^2$. 5

(c) Prove that two tangents can be drawn from a given point to an ellipse. 5

UNIT—III

5. *(a)* Show that the equation of the plane which passes through the point (2, −3, 8) and is normal to the line joining the points (3, 4, −1) and (2, −1, 5) is $x+5y-6z+61=0$. 5

(b) Find the equation to the locus of the point whose distance from the origin is equal to its perpendicular distance from the plane $2x+3y-6z=9$. 5

(c) Find the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

and $$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

Also show that the lines are coplanar. 5

6. *(a)* Prove that the plane

$$2x-2y+z+12=0$$

touches the sphere

$$x^2+y^2+z^2-2x-4y+2z=3$$ 5

(b) Find the equation of the right circular cone, whose vertex is (3, 2, 1), axis is the line

$$\frac{x-3}{4}=\frac{y-2}{1}=\frac{z-1}{3}$$

and semi-vertical angle is 30°. 5

(c) Find the equation of the cylinder generated by the lines parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{5}$, the guiding curve being the conic $x=0,\ y^2=8z$. 5

UNIT—IV

7. *(a)* If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$, show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs and $|\vec{b}|=1,\ |\vec{c}|=|\vec{a}|$. 5

(b) Show that the four points $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar if

$$[\vec{b}\ \vec{c}\ \vec{d}]+[\vec{c}\ \vec{a}\ \vec{d}]+[\vec{a}\ \vec{b}\ \vec{d}]=[\vec{a}\ \vec{b}\ \vec{c}]$$ 5

(c) Show that the volume of the parallelopiped whose edges are represented by $(3i+2j-4k)$, $(3i+j+3k)$ and $(i-2j+k)$ is 49 cubic units. 5

8. (a) Show that a necessary and sufficient condition for a vector $\vec{u}(t)$ to have a constant direction is $\vec{u} \times \frac{d\vec{u}}{dt} = 0$. 6

(b) Find the unit tangent vector at any point on the curve $x = a\cos t$, $y = a\sin t$, $z = bt$. 4

(c) If $\vec{r} = 3t\hat{i} + 3t^2\hat{j} + 2t^3\hat{k}$, then find

$$\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}$$ 5

UNIT—V

9. (a) Find the directional derivative of

$$\phi = (x^2 + y^2 + z^2)^{-1/2}$$

at the point $P(3, 1, 2)$ in the direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$. 5

(b) Show that

$$\text{grad}\, e^{(x^2+y^2+z^2)} = 2e^{r^2}$$

where $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$. 5

(c) If

$$\vec{A} = (x + y + 1)\hat{i} + \hat{j} + (-x - y)\hat{k}$$

prove that $\vec{A} \cdot (\nabla \times \vec{A}) = 0$. 5

10. (a) Prove that

$$\text{div}\, \hat{r} = \frac{2}{r}$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$ 5

(b) Find the equations of the tangent plane and normal line to the surface

$$f = 2xz^2 - 3xy - 4x - 7$$

at the point (1, −1, 2). 6

(c) Find the gradient and unit vector normal to the surface $f = x^2 + y - z$ at the point (1, 0, 0). 4

★★★